



Review Article

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## An Alternative Method for Outgoing Quality with Double Sampling Plan

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### ABSTRACT

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In this study, we have been developed a minimum variance and VOQL double sampling plan. We have minimized the variance of outgoing quality to develop a sampling plan under total rectification. This is an improvement over AOQL, which is commonly used in acceptance sampling plan. The thrust of this effort is to establish criteria for minimum variance sampling plans and derive the techniques for their determination. The result are explained & discussed and shown through the various tables.

### Introduction

When a series of lots, produced by a random process (assumed to be under statistical control), are submitted, the acceptance sampling ensures a specified risk of accepting lots of given quality, are thus yields quality assurance. Often, in practice, an acceptance-sampling plan is followed by further inspection of lots, when the lots are rejected by the inspection plan. These programs referred to as “rectifying inspection plans” give a definite assurance regarding the quality of the lots passed by the program. For a good account of double and single sampling inspection plans, the reader is referred to (4). Most of the rectifying inspection plans for lot by lot sampling call for 100 percent inspection of the lots, where all the non-

conforming items found during the sampling and rectifying inspection are replaced by good ones.

Some important features of rectifying inspection program are “Average outgoing Quality” (AOQ) and the “Average Outgoing Quality Limit” (AOQL), the maximum value of AOQ (4) have designed sampling plans having the specified AOQL and minimizing the ATI for a given “Process average”.

Although AOQ and AOQL are the salient features of the quality of the outgoing lots, they seldom reflect the lot to lot variations in OQ (Outgoing Quality). For instance, as point out in (11), a sampling plan may exist which has an adequate AOQ at a given value of the process average  $p$ , but whose corresponding

variance of outgoing quality could be considerably large. This may cause a considerable departure of the actual quality of the delivered lots from the value of AOQ.

Therefore, AOQ alone is a misleading measure of the effectiveness of the sampling plan. We should also take into account the variance of OQ in assessing a sampling plan. For single sampling rectification inspection plans (11) introduce the OQ as explicit the random variable and based on the variance of OQ, he derived the minimum variance single sampling plans. Analogous to the design of a rectifying inspection plan with a given AOQL (5), (P.302), he devised also plans with designated VOQL (Variance of outgoing quality limit), the maximum variance of OQ.

Our aim in this paper is to obtain the minimum variance and VOQL double sampling plans. In section 2, we lay down the basic assumptions, and derive the distributions of OQ for a double sampling plan. In section 3, we develop the sampling plans, under total rectification, which minimize the variance of the outgoing quality. Finally, in section 4, we obtain VOQL plans, which have the specified maximum variance of the outgoing quality.

**Basic assumptions and distributional properties of OQ of a double sampling plan**

Suppose we have a random process, operating in a random manner but under statistical control, which turns out (on the average) 100p percent non-conforming items. The product of this process will be said to be of quality p or the product is said to have process average p. If lots of size N are made up of thus products, then the number of non-conforming units of the lots will follow a binomial distribution or, in order words the lot quality is binomial variate with parameters N and p, the process average. Assume lots of size N, and of quality

p, are submitted for inspection. Our concern here is to employ suitable double sampling (rectification) plan to make a decision regarding the acceptance of lots.

**Notations and basic assumptions**

We employ the following notations:

$n_1, n_2$ : size of the first and that of the second sample.

$c_1, c_2$ : acceptance number for the first sample of size  $n_1$  and for the

combined sample of size  $n_1 + n_2$ .

$X_i$ : number of non-confirming units in the i-th sample  $i = 1, 2, 3, \dots$

$b(x | n, p)$ :  $\binom{n}{x} p^x (1 - p)^{n-x}$ , for  $x = 0, 1, 2, 3, \dots, n$ .

$P_{a1}, P_{a2}$ : the probability of accepting the lot based on the first sample, and the one based on second sample.

$S_{N-n}, S_{N-n_1-n_2}$ : the number of non-conforming items in the remaining portion of the lot when the first sample of size  $n_1$  and when the second sample of size  $n_2$  is taken from the lot

$X$ : the number of non-conforming items in the lot of size N drawn at random from a (theoretically infinite) random process.

$h(x | n, X, N)$ : the probability mass function of a hyper-geometric distribution and is equal to

$$\frac{\binom{X}{x} \binom{N-X}{n-x}}{\binom{N}{n}} = \frac{\binom{n}{x} \binom{N-n}{X-x}}{N}$$

Where  $N = 0, 1, 2, \dots, X = 0, 1, \dots, N$ ;  $n = 0, 1, \dots, N$  and  $\max(n - N + X, 0) \leq x \leq \min(X, n)$

$Y_1$ : a discrete random variable assuming three values, say 0, 1 or c accordingly as the lot is rejected, accepted or no decision is taking on the basis of the first sample.

$Y_2$ : also on indicator variable taking the value zero or unity respectively when the lot is finally rejected or accepted on the basis of both the samples.

Our basic assumptions are the following:

- (a)  $P(X = x) = b(x | N, p)$ ,  $x = 0, 1, \dots, N$ , denotes the prior distribution of  $X$ .
- (b)  $P(X_1 = x_1 | X) = h(x_1 | n_1, X, N)$  and
- (c)  $P(X_2 | X, X_1) = h(x_2 | n_2, X - X_1, N - X_1)$

With the above notations and assumptions, we have the following results:

(i) The joint distribution of  $X, X_1$  and  $X_2$  is  $P(X = x, X_1 = x_1, X_2 = x_2) = b(x | N, p) h(x_1 | n_1, x, N) h(x_2 | n_2, x - x_1, N - x_1) = b(x_1 | n_1, p) b(x_2 | n_2, p) b(x - x_1 - x_2 | N - n_1 - n_2, p)$  which shows that the random variable  $X_1, X_2$  and  $X - X_1 - X_2$  are independent binomial random variables with the same parameter  $p$  (see, for example, Hald (1981))

(ii)  $Pa_1 = \sum_{x=0}^{c_1} b(x | n_1, p)$ ,  $Pa_2 = \sum_{x=c_1+1}^{c_2} b(x | n_1, p) B(c_2 - x | n_2, p)$  where  $B(|n, p)$  denotes the binomial distribution function with parameters  $n$  and  $p$ .

(iii)  $S_{N-n_1}$  is binomial variate with parameters  $N - n_1$  and  $p$  and is independent  $Y_1$  and  $Z$ .

Similarly  $S_{N-n_1-n_2}$  also has a binomial distribution with parameters  $N - n_1 - n_2$  and  $p$  and is independent of  $Y_1, Z$  and  $Y_2$ .

(iv)  $P(Y_1 = j) = Pa_1, Pr_1, j = 0, 1; P(Y_1 = c) = 1 - Pa_1 - Pr_1 = Pc$  (say) when  $Pr_1 = 1 - B(C_2 | n_1, p)$  is the probability of rejection based on the first sample and  $Pc$  is the probability of continuation to the second sample.

(v) Define  $Z = 1 - I(Y_1 = c)$ , where  $I(A)$  denote the indicator function of the set  $A$ . Then

- a)  $P(Z = 0) = Pc, P(Z = 1) = Pa_1 + Pr_1$  and
- (b)  $P(Y_2 = 1 | Z = 0) = Pa_2 / Pc, P(Y_2 = 0) = Pr_2 / Pc$  Where  $Pr_2 = (1 - pa_1 - Pr_1 - pa_2)$  denote the probability of final rejection.

In the theoretical analysis that follows, we assume for simplicity that 100 percent inspection of the rejected lot is perfect.

### Distribution of OQ

We define the outgoing quality, associated with a doubling sampling rectification plan, as the quality of the material turned out by the combinations of sampling and 100 percent inspection.

That is, the random variable OQ is defined as

$$OQ = \frac{\{S_{N-n_1} + S_{N-n_1-n_2}(1 - Y_1)\}Y_1Z + S_{N-n_1-n_2}Y_2(1 - Z)}{N} \dots(2.1)$$

Where all the random variables of the right hand side of (2.1) are defined in section 2.1

Observe that  $N.OQ$  is a discrete random variable taking the values  $0, 1, 2, \dots, N - n_1$ . The probability distribution of OQ can be seen to be

$$P(OQ = j / N) = \begin{cases} Pa_1(1 - P)_{N-n_1} + Pa_2(1 - p)^{N-n} + (1 - Pa_1), & \text{if } j = 0 \\ Pa_1 b(j | N - n_1, p) + Pa_2 b(j | N - n, p) & \text{if } j = 1, \dots, N - n \\ Pa_1 b(j | N - n_1, p) & \text{if } j = N_1 - n + 1, \dots, N - n \end{cases}$$

Where  $n = n_1 + n_2$  and  $Pa = pa_1 + Pa_2$ . The expectation of OQ denoted by AOOQ, is given by  $NAOOQ = E(S_{N-n_1} Y_1 Z) + E(S_{N-n} Y_1 (1 - Y_1)Z) + E(S_{N-n} Y_2 (1 - Z)) = E(S_{N-n_1}) E(Y_1 Z) + E(S_{N-n}) \{E(Y_1^2 Z)\} + E(S_{N-n} Y_2 (1 - Z)) = E(S_{N-n}) E(Y_1 Z) + E(S_{N-n}) E(Y_1 Z (1 - Y_1)) + E(S_{N-n}) E(Y_2 (1 - Z)) \dots(2.2)$

Here, we know that  $S_{N-n_1}$  are the non-confirming items in the lot when we take first sample of size  $n_1$ .

Then,  $E(S_{N-n_1}) = N-n_1$   
 Similarly,  $E(S_{N-n}) = N-n$   
 $E(Y_1 Z)$  = the probability of acceptance on the basis of first sample.  
 $= pPa_1 \dots\dots\dots(2.3)$

$E(Y_1 (1-Y_1)Z) = 0$  when  $Y_1 = 0$  or  $1$   
 $E(Y_2 (1-Z)) = Y_2$  takes value 0 or 1 only.  
 When  $Y_2 = 1$   
 $E(Y_2 (1-Z)) = p Pa_2 \dots\dots\dots(2.4)$

On putting equation (4.2.2.3), we get  
 $N.E(OQ) = (N-n_1) p Pa_1 + 0 + (N-n) p Pa_2$   
 $= (N-n_1) p Pa_1 + (N-n) p Pa_2 = ((N-n_1) Pa_1 + (N-n) Pa_2) p \dots\dots(2.5)$

Similarly,

$$N^2E(OQ^2) = E(\{S_{N-n_1} + S_{N-n}(1-Y_1)\} Y_1 Z + S_{N-n} Y_2 (1-Z))^2 = E(S_{N-n_1}^2 Y_1^2 Z^2 + S_{N-n}^2 (1-Y_1)^2 Y_1^2 Z^2 + S_{N-n}^2 Y_2^2 (1-Z)^2 + 2 S_{N-n_1} S_{N-n} (1-Y_1) Y_1 Z + 2 S_{N-n}^2 (1-Y_1) Y_1 Z Y_2 (1-Z) + 2 S_{N-n_1} S_{N-n} Y_1 Y_2 Z (1-Z))$$

The terms which have  $Y_1 (1-Y_1)$  will be zero because  $Y_1$  takes value of 0 or 1.

$$N^2(E(OQ^2)) = E(S_{N-n_1}^2 Y_1^2 Z^2 + S_{N-n}^2 Y_2^2 (1-Z)^2 + 2 S_{N-n_1} S_{N-n} Y_1 Z Y_2 (1-Z))$$

In this equation the terms expectation will be zero because  $Y_1 Z$  and  $Y_2(1-Z)$  are independent and the expectations of independent terms will be covariance of independent terms will be zero.

$$N^2E(OQ^2) = E(S_{N-n_1}^2 Y_1^2 Z^2 + S_{N-n}^2 Y_2^2 (1-Z)^2) = E(S_{N-n_1}^2) E(Y_1^2 Z^2) + E(S_{N-n}^2) E(Y_2^2 (1-Z)^2) = Pa_1 E(S_{N-n_1}^2) + Pa_2 E(S_{N-n}^2) = Pa_1 ((N-n_1) p(1-p) + (N-n_1)^2 p^2) + Pa_2 ((N-n) p(1-p) + (N-n)^2 p^2) = Pa_1 (N-n_1) p(N-n_1) p^2 + (N-n_1)^2 p^2 + Pa_2 ((N-n) p - (N-n) p^2 + (N-n)^2 p^2)$$

$$N^2E(OQ^2) = Pa_1 ((N-n_1)p + (N-n_1) (N-n_1-1)p^2) + Pa_2 ((N-n)p + (N-n) (N-n-1)p^2)$$

Using the fact  $EW^2 = n(n-1)p^2 + np$ , if  $W$  is a binomial variate with parameters  $n$  and  $p$ , therefore, the variance of  $OQ$ , denoted by  $VOQ$ , can be seen to be

$$N^2 VOQ = N^2E(OQ^2) - (N.AOQ)^2 = Pa_1 \{(N-n_1) (N-n_1-1)p^2 + (N-n_1)p\} + Pa_2 \{(N-n) (N-n-1)p^2 + (N-n)p\} - (N\{N-n_1\}Pa_1 + (N-n)Pa_2) p^2 = p^2 ((N-n_1)^2 Pa_1(1-pa) + (N-n)^2 Pa_2 (1-Pa) + n_1^2 Pa_1 Pa_2) + p(1-p) (N-n)pa + nPa_1$$

**Minimum variance double sampling plans**

**Properties of admissible and minimum variance plan**

It is apparent that the  $AQO$  is a function of the process average  $p$ . Suppose we are interested in the  $AQO$  at a specific value of  $p$ , say  $p_0$ , which may be dictated from practical considerations. Let  $AOQ_0$ ,  $VOQ_0$ ,  $Pa_{10}$ ,  $Pa_{20}$  and  $Pa_0$  be the respective quantities calculated at  $p_0$ . Also let  $SOQ_0$  denote the positive square root of  $VOQ_0$ .

Our aim now is to find a plan, which minimize  $VOQ_0$  and satisfies the conditions:

$$AOQ_0 = \left\{ \left( \frac{N-n_x}{N} \right) Pa_0 + \frac{n^2}{N} Pa_{10} \right\} p_0, \dots\dots\dots(3.1.1)$$

and  $Pa_0 \leq M \dots\dots\dots(3.1.2)$

Where  $M$  is a specified constant less than 1.

The condition (4.3.2.1) is simple but a critical assumption (See (11) p. 557) in order to obtain meaningful optimal plans. For a plan with  $Pa = 1$  will always lead to the acceptance of the lot irrespective of its quality. Note also that the value of  $M$  could be specified in advance by the consumer or the experimenter

depending upon the value of  $p_0$  and his requirements. Observe first that from (4.3.1.1) and (4.3.1.2) that

$$M \geq \left[ \frac{N}{N - n_1} \right] \left[ \frac{AOQ_0}{p_0} \right] + \left[ \frac{n_2}{N - n_1} \right] Pa_{20} \dots\dots\dots(3.1.3)$$

We know express  $VOQ_0$  in terms of the specified  $AOQ_0$  and  $p_0$  substituting (3.1.1) in (2.2.3). We obtain

$$VOQ_0 = \frac{p_0^2}{N^2} \left[ (N - n_1) 2Pa_{10} + (N - n)^2 Pa_{20} \right] + \frac{(1 - p_0)}{N} AOQ_0 - AOQ_0^2 \dots\dots\dots(3.1.4)$$

There are usually many plans satisfying (3.1.1) and (3.1.2), which we call admissible plans, for a given  $AOQ_0$ ,  $N$  and  $P_0$ . Among these plans, the one which minimizes  $VOQ_0$  or equivalently  $(N - n_1)^2 Pa_{10} + (N - n)^2 Pa_{20}$  is called an optimal double sampling plan.

**Determination of minimum variance plan**

As discussed in the earlier section, our first step is to solve the equation (3.1.1) for  $n_1$  and  $n_2$  for some selected values of  $c_1$  equation (3.1.1) for  $c_1$  and  $c_2$ . The determination of an arbitrary double sampling plan satisfying (3.1.1) for the given values of  $AOQ_0$ ,  $p_0$  and  $N$  is complicated. Therefore we consider the case where  $n_2$  is equal to a constant multiple of  $n_1$ .

We employ Poisson approximation to the binomial distribution in the calculation of probabilities involved in a double sampling plan.

We obtain double sampling plans satisfying (3.1.1) for the sets of chosen values of  $c_1$  and

$c_2$ , and for the cases of practical interest  $n_2 = n_1 = n_0$ , the equation (3.1.1) reduce to

$$(Np_0 - Z_0) C^{-Z_0} \left[ \sum_{i=1}^{c_1} \frac{Z_0^i}{i!} \right] + (NP_0 - 2Z_0) e^{-Z_0} \sum_{i=c_1+1}^{c_2} \frac{Z_0^i}{i!} \left[ \sum_{j=0}^{c_2-i} \frac{Z_0^j}{j!} \right] = NAOQ_0 \dots\dots\dots(3.2.1)$$

Similarly, when  $n_2 = 2n_1 = 2n_0$ , equation (3.1.1) becomes

$$(Np_0 - Z_0) C^{-Z_0} \left[ \sum_{i=1}^{c_1} \frac{Z_0^i}{i!} \right] + (NP_0 - 3Z_0) e^{-Z_0} \sum_{i=c_1+1}^{c_2} \frac{Z_0^i}{i!} \left[ \sum_{j=0}^{c_2-i} \frac{2Z_0^j}{j!} \right] = NAOQ_0 \dots\dots\dots(3.2.2)$$

Let us consider first the case  $n_1 = n_2 = n_0$  Let  $p_0 = 0.02$ ,  $N = 1,000$  and  $AOQ_0 = 0.015$ . For certain chosen values of  $c_1$  and  $c_2$ , we solve the non linear equation (3.2.1) for  $Z_0$  using the method of bisection. Then the required  $n_0$

$$= \left[ \left( \frac{Z_0}{p_0} + 1 \right) \right],$$

where  $x$  denotes the integral part of  $x > 0$ . We repeat the above procedure for different values of  $c_1$  and  $c_2$  and obtain the plans (satisfying (3.1.1)) for which the calculated values  $Pa_0$ ,  $SOQ_0$ ,  $AOQL$  and  $AOQ_0$  are also given in Table 1.

Similarly the sample plans for the case  $n_2 = 2n_1 = 2n_0$ , have been calculated by solving the equation (3.2.2). These plans and their associated characteristics are given in Table 2.

Consider now the problem of determining the minimum variance sampling plans for the given values of say,  $N = 1000$ ,  $p_0 = 0.002$ ,  $APQ_0 = 0.015$  and  $M = 0.95$ . For the case of equal sample sizes (Table 1), the plans starting from  $(c_1, c_2) = (0, 1)$  to  $(c_1, c_2) = (5, 10)$  are admissible plans. The optimal plan

(179, 179, 5, 10) with  $SOQ_0 = 0.005645$  has about 37% reduction over 0.008912, the maximum value of  $SOQ_0$  of admissible plans. Similarly, for the case  $n_2 = 2n_1 = 2n_0$  (Table 2), the optimal plan (169, 169, 5, 10) has about 34% reduction over the maximum attainable  $SOQ_0$ .

Finally, we remark that the information regarding the AOQL as also provide for all the plans listed in the tables. This would help the experimenter to choose a minimum variance plan with acceptable levels of AOQL. Also, note that, from the list column of the table 1 and 2, the actual value (or the calculated) of  $AOQ_0$  of all the plans is practically the same as the designed one.

**Double sampling VOQL plans**

In this section we develop double sampling plans which have the designated VOQL, the maximum variance of the outgoing quality. Such plans are called VOQL plans, similar to AOQL plans available in the literature (4). (11) devised VOQL plans for single sampling (7) proposed a procedure for finding a double sampling (non-rectifying) plans such that the probability of accepting the lot is at least  $1 - \alpha$ , if  $p = p_0$  and at most  $\beta$  if  $p = p_1 (> p_0)$

When the lot size  $N$  is large, and the terms with coefficients of order  $o(N^{-1})$  and  $o(N^{-2})$  are ignored, we have from (4.2.2.3)

$$VOQ \approx \frac{p^2}{N^2} [(N - n_1)^2 p_0 (1 - p_0)] \dots\dots\dots(4.1)$$

Where  $P_a = Pa_1 + Pa_2$ . Setting  $n_1 p = z_1$  and  $n_2 p = z_2$ , we have

$$VOQ \approx \left[ \frac{N - n_1}{N n_1} \right] z_1^2 p_a (1 - p_a) \dots\dots\dots(4.2)$$

Where now,

$$P_a = \sum_{i=0}^{c_1} \frac{e^{-z_1} z_1^i}{i!} + \sum_{j=c_1+1}^{c_2} \frac{e^{-z_1} z_1^i}{i!} \left[ \sum_{j=0}^{c_2-j} \frac{e^{-z_2} z_2^j}{j!} \right] \dots\dots\dots(4.3)$$

Let now,

$$VOQ F (c_1, c_2, z_1, z_2) = z_1^2 P_a (1 - P_a) \dots\dots\dots(4.4)$$

Our aim is to find  $n_1, n_2$  for chosen  $c_1, c_2$ , such that  $VOQ \leq VOQL$ , a specified quantity. This lead us to find  $VOQ F = \max_{z_1, z_2} VOQ F (z_1, z_2)$ . For may choices of  $c_1$  and  $c_2$  it is observed that VOQF corresponds to the case  $Z_2 = 0$ . To avoid this situation, we impose the constraint

$$z_2 \geq k z_1 (k > 0) \text{ and define}$$

$$VOQ F_k = \max VOQ F (z_1, z_2) \dots\dots\dots(4.5)$$

$$z_2 \geq k z_1$$

It can be seen that the maximum in the right hand side of (4.5) attains at  $z_2 = k z_1$  and observed also that, from (4.2) and (4.4), we have for given VOQL.

$$VOQL = \left[ \frac{N - n_1}{N n_1} \right]^2 VOQ F_k$$

Which, solving for  $n_1$ , gives  $n_1 = \frac{N \sqrt{VOQ F_k}}{N \sqrt{VOQL} + \sqrt{VOQ F_k}} \dots\dots\dots(4.6)$

$$n_2 = k n_1 \dots\dots\dots(4.7)$$

The procedure for finding VOQL plans is given below. Here  $VOQL_0$  denote the calculated value of VOQL for a particular plan.

(1) For chosen  $c_1$  and  $c_2$  and  $k$ , compute VOQL  $F_k$  from (4.5)

(2) Compute  $n_1$  and  $n_2$  using (4.6) and (4.7) and round them to the next smallest integer.

(3) For the plan  $(c_1, c_2, n_1, n_2)$ , determined from step 2, compute  $VOQL_0$ . If  $VOQL_0 \leq VOQL$ , go the step 4. Otherwise set  $n_1 = n_1 + 1$  and  $n_2 = k(n_1 + 1)$  and repeat this step.

(4) Change the values of  $c_1$  and / or  $c_2$ , and repeat the steps 1–3

The VOQL plans for the cases  $k = 1$  and  $k = 2$  are respectively given in tables 3 and 4 for the case  $N = 1000$  and a  $VOQL = 0.000225$ .

Let for example,  $p_0 = 0.01$ . Then the VOQL plans for the case  $k = 1$  and  $k = 2$ , denoted by \* \* in tables, are respectively given by (90, 90, 1, 4) and (80, 160, 1, 5).

**Table.1**

$C_1$	$C_2$	$n_0$	$P_{a0}$	$SOQ_0$	$AOQL_0$	Calculated $AOQ_0$
0	1	26	0.7783	0.0089	0.0187	0.0150
0	2	41	0.7951	0.0085	0.0166	0.0150
1	2	51	0.7960	0.0085	0.0171	0.0150
1	3	63	0.8122	0.0082	0.0162	0.0150
1	4	76	0.8354	0.0072	0.0159	0.0150
2	3	79	0.8164	0.0081	0.0165	0.0150
2	4	88	0.8274	0.0078	0.0159	0.0150
2	5	98	0.8499	0.0074	0.0158	0.0150
3	4	107	0.8431	0.0075	0.0164	0.0150
3	5	113	0.8514	0.0074	0.0160	0.0150
3	6	121	0.8672	0.0070	0.0159	0.0150
3	7	132	0.8807	0.0067	0.0156	0.0150
4	7	145	0.8845	0.0067	0.0159	0.0150
4	8	153	0.8983	0.0064	0.0158	0.0150
4	9	160	0.9188	0.0060	0.0159	0.0150
5	8	167	0.9075	0.0062	0.0163	0.0150
5	11	173	0.9183	0.0059	0.0162	0.0150
5	10	179	0.9331	0.0056	0.0162	0.0150
5	12	191	0.9629	0.0050	0.0166	0.0150
6	12	203	0.9583	0.0050	0.0166	0.0150
6	14	211	0.9803	0.0046	0.0172	0.0150
7	16	226	0.9914	0.0042	0.0181	0.0150
8	17	236	0.9945	0.0041	0.0188	0.0150
9	20	243	0.9992	0.0039	0.0209	0.0150

**Table.2**

<b>C<sub>1</sub></b>	<b>C<sub>2</sub></b>	<b>n<sub>0</sub></b>	<b>P<sub>a0</sub></b>	<b>SOQ<sub>0</sub></b>	<b>AOQL<sub>0</sub></b>	<b>Calculated AOQ<sub>0</sub></b>
<b>0</b>	1	21	0.7762	0.0090	0.0195	0.0151
<b>0</b>	2	31	0.7781	0.0087	0.0166	0.0149
<b>0</b>	3	40	0.8141	0.0082	0.0162	0.0150
<b>1</b>	2	46	0.7919	0.0086	0.0177	0.0150
<b>1</b>	3	52	0.8029	0.0084	0.0166	0.0150
<b>1</b>	4	60	0.8153	0.0081	0.0159	0.0150
<b>2</b>	4	78	0.8201	0.0080	0.0165	0.0150
<b>2</b>	5	83	0.8294	0.0078	0.0160	0.0150
<b>2</b>	6	89	0.8443	0.0076	0.0157	0.0150
<b>3</b>	5	106	0.8431	0.0075	0.0164	0.0150
<b>3</b>	6	109	0.8469	0.0075	0.0161	0.0150
<b>3</b>	7	113	0.8559	0.0073	0.0158	0.0150
<b>3</b>	8	117	0.8725	0.0070	0.0158	0.0150
<b>4</b>	7	136	0.8685	0.0070	0.0162	0.0150
<b>4</b>	8	138	0.8745	0.0069	0.0160	0.0150
<b>4</b>	9	140	0.8868	0.0067	0.0160	0.0150
<b>5</b>	10	164	0.9032	0.0063	0.0163	0.0150
<b>5</b>	11	166	0.9121	0.0061	0.0162	0.0150
<b>5</b>	12	169	0.9220	0.0060	0.0161	0.0150
<b>5</b>	15	178	0.9591	0.0054	0.0162	0.0150
<b>6</b>	16	196	0.9632	0.0052	0.0165	0.0150
<b>6</b>	20	202	0.9919	0.0048	0.0171	0.0150
<b>6</b>	24	203	0.9993	0.0046	0.0181	0.0150
<b>7</b>	06	218	0.9996	0.0045	0.0186	0.0150
<b>8</b>	30	230	0.9999	0.0043	0.0196	0.0150
<b>9</b>	32	239	0.9999	0.0041	0.0206	0.0150
<b>10</b>	32	245	0.9999	0.004	0.0216	0.0150

Advisable double sampling plans ( $n_2 = 2n_1 = 2n_0$ ) for  $AOQ_0 = 0.015$   
 $P_0 = 0.020$ ,  $N = 1000$ ,  $0.776 \leq M \leq 0.999$  and  $ATI_0 = 250$



**Table.3**

<b>c</b>	<b>c<sub>2</sub></b>	<b>n<sub>0</sub></b>	<b>VOQF<sub>1</sub></b>	<b>VOQL<sub>C</sub></b>
0	1	49	0.5753	.000218
0	1	49	0.8570	.000213
0	3	71	1.3011	.000199*
0	4	84	1.8897	.000193
0	5	98	2.6153	.000183
1	2	77	1.5096	.000219
1	3	81	1.7347	.000215
1	4	90	2.1728	.000204**
1	5	101	2.7992	.000195
1	6	113	3.5912	.000185
2	3	104	2.9578	.000220
2	4	106	3.1013	.000215
2	5	111	3.4576	.000207*
2	6	119	4.0482	.000199
2	7	129	4.8487	.000189
3	4	129	4.9190	.000223
3	5	130	4.9993	.000220
3	6	133	5.2450	.000211*
3	7	138	5.7332	.000202
3	8	146	6.4740	.000193
4	5	154	7.3879	.000220
4	6	154	7.4294	.000221
4	7	156	7.5796	.000214*
4	8	159	7.9345	.000206
4	9	164	8.5549	.000194
5	6	177	10.3607	.000222
5	7	177	10.3811	.000222
5	8	178	10.4556	.000219*
5	9	180	10.6974	.000213
5	10	183	11.1655	.000206
6	7	199	13.8354	.000224
6	8	199	13.8450	.000224
6	9	200	13.8899	.000220

<b>c<sub>1</sub></b>	<b>c<sub>2</sub></b>	<b>n<sub>0</sub></b>	<b>VOQF<sub>1</sub></b>	<b>VOQL<sub>C</sub></b>
6	10	200	14.0293	.000221*
6	11	202	14.3506	.000226
6	12	205	14.9345	.000209
6	13	210	15.8195	.000197
7	8	221	17.8108	.000222
7	9	221	17.8152	.000222
7	10	221	17.8381	.000222
7	11	221	17.9171	.000223*
7	12	222	18.1210	.000220
7	13	224	18.5375	.000215
7	14	227	19.2388	.000208
8	10	241	22.2886	.000225
8	11	241	22.2999	.000225
8	12	241	22.3426	.000225*
8	13	242	22.4644	.000222
8	14	242	22.7405	.000223
8	15	244	23.2562	.000218
9	12	261	27.2688	.000224
9	13	261	27.1912	.000224
9	14	261	27.3606	.000224
9	15	261	27.5329	.000225*
9	16	262	27.8872	.000222
9	17	263	28.5051	.000220
9	18	266	29.4447	.000214
9	19	270	30.7271	.000205
10	14	280	32.7528	.000224
10	15	280	32.7909	.000224
10	16	280	32.8935	.000224
10	17	280	33.1232	.000224
10	18	280	33.5603	.000225*
10	19	281	34.1830	.000222
10	20	284	35.3430	.000215
10	21	288	36.7579	.000206
10	22	293	38.5188	.000195
10	23	299	40.6037	.000182

VOQL plans ( $n_1 = n_2 = n$ ) for  $VOQL_0 = 0.000225$  and  $N = 1000$

**Table.4**

<b>c<sub>1</sub></b>	<b>c<sub>2</sub></b>	<b>n<sub>0</sub></b>	<b>VOQF<sub>2</sub></b>	<b>VOQL<sub>C</sub></b>
0	1	90	0.4881	.000222
0	2	96	0.5519	.000215
0	3	108	0.7067	.000197
0	4	122	0.9403	.000194*
0	5	140	1.2412	.000182
1	2	152	1.4559	.000220
1	3	152	1.4727	.000221
1	4	154	1.5378	.000219
1	5	160	1.6943	.000209**
1	6	172	1.9528	.000196
2	3	206	2.3904	.000224
2	4	206	2.9342	.000224
2	5	206	2.9527	.000225
2	6	208	3.0126	.000220
2	7	212	3.1541	.000213*
2	8	220	3.4050	.000200
2	9	230	3.7655	.000191
3	5	258	4.9069	.000223
3	6	258	4.9115	.000223
3	7	258	4.9293	.000223
3	8	260	4.9816	.000218
3	9	262	5.1026	.000215*
3	10	268	5.3275	.000202
3	11	276	5.6727	.000192
4	7	308	7.3834	.000220
4	8	308	7.3880	.000220
4	9	308	7.4040	.000220
4	10	308	7.4480	.000221*
4	11	310	7.5482	.000217
4	12	314	7.7401	.000209
5	8	354	10.3585	.000222
5	9	354	10.3596	.000222
5	10	354	10.3640	.000222
5	11	354	10.3777	.000222

<b>c<sub>1</sub></b>	<b>c<sub>2</sub></b>	<b>n<sub>0</sub></b>	<b>VOQF<sub>2</sub></b>	<b>VOQL<sub>C</sub></b>
5	12	356	10.4138	.000218
5	13	356	10.4952	.000218
6	11	398	13.8357	.000224
6	12	398	13.8396	.000224
6	13	398	13.8510	.000224
6	14	398	13.8803	.000224*
6	15	400	13.9453	.000220
6	16	402	14.0773	.000216
6	17	404	14.2965	.000213
7	15	442	17.8244	.000222
7	16	442	17.8477	.000222
7	17	442	17.8992	.000222
7	18	442	18.0011	.000222*
7	19	444	18.1828	.000219
7	20	446	18.4755	.000215
7	21	450	18.9029	.000209
8	18	482	22.3164	.000225
8	19	482	22.3568	.000225
8	21	482	22.5826	.000224*
8	22	484	22.8235	.000221
8	23	488	23.1886	.000215
8	24	492	23.6980	.000208
9	22	522	27.3804	.000224
9	24	522	27.6903	.000224
9	25	522	27.9946	.000224*
9	26	526	28.4344	.000217
9	27	530	29.0265	.000211
9	28	534	29.7767	.000205
10	22	560	32.7573	.000224
10	25	560	32.9205*	.000224
10	27	560	33.3245	.000224
10	28	560	33.6953	.000223
10	29	562	34.2114	.000220
10	30	566	34.8869	.000214

Hence concluded in this paper, a double acceptance sampling plan have been developed which is based on the OQ specifications. It has been noticed that no DASP consider variance criteria except a few for OQ. We have been presented various values for DASP parameters and the necessary tables based on the VOQL plan. However, the VOQL sampling plan is very sensitive to the product quality. The different VOQL plan aspects have been discussed. These aspects have their combined effect to get an economic sampling plan with VOQL concept in decision making analysis.

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