

Original Research Article

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Statistical Models to Predict the Height of Trees and Yield of Field Crop in the Agroforestry System

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ABSTRACT

In this investigation an effort is made to fit a model for the height of different tree species and the yield of field crop. Here we have considered three trees species namely *Ceiba pentandra*, *Terminalia bellarica*, *Mangifera indica* and soybean as field crop. The required data was collected from AICRP on agroforestry, UAS Dharwad. The different models were tried and the best two models were selected based on R^2 and Standard error values. The best fitted models for height of *Ceiba pentandra*, *Terminalia bellarica* and *Mangifera indica* were MMF model followed by Gompertz relation, MMF model followed by Weibull model and MMF model followed by polynomial. The results indicated that MMF model was found better for all the tree species. To predict the yield of field crop (soybean, at 1m distance) was increased in the beginning i.e. up to five years and then declining trend was notice. The best fitted models for the yield of soybean were, Rational model followed by Sinusoidal model in case of *Ceiba pentandra*, Rational model followed by Hoerl model for *Terminalia bellarica*, and Hoerl model followed by Rational model for *Mangifera indica*.

Keywords

Morgan-Mercer-Flodin (MMF), Gompertz, Weibull, Sinusoidal, Hoerl, Rational etc.

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Introduction

The productive area available in the country is around 300 m.ha of which 76.5 m.ha is recorded forest area. In percentage, India's Forest Cover accounts for 20.6% of the total geographical area of the country as of 2005. Actual forest cover is 63.34 million ha of which 26.13 million ha is degraded. About 20 million ha is covered under private tree planting (agroforestry, farm forestry, social forestry and other plantations). In order to meet the increasing demand of our fast growing population, we would require to

boost the production of food grain, fuel wood and also green and dry fodder for livestock to the tune of 1061mt and 589mt, respectively, besides 75 million m^3 of timber. Thus, to meet the gap between demand and supply of fodder, fuel wood and timber, the emphasis should be on the integration of tree component with agricultural crops, which is precisely agroforestry.

Growth refers to the increase in dimensions of one or more individuals in a forest stand over a given period of time (e.g. volume growth in m^3 /ha/ y). Yield refers to their final

dimensions at the end of a certain period (e.g. volume in m³/ha).

A statistical model is a set of mathematical equations which describes the behaviour of an object of study in terms of random variables and their associated probability distributions.

Modelling the goal of modelling is to understand reality mathematically. A model is considered a simplified representation of reality. Many complex interactions and results are depicted with simplicity to reach a decision. It is expected to be a good representation of those factors that influence production and management. Modeling has been used in natural sciences since centuries in one way or the other. Models have been built in the physical, biological and social sciences.

Various researches have defined modeling in their own way viz.,

“Modeling is described as a representation of our so called ‘real world’ in mathematical terms”, so that we may gain a more precise understanding of its significant properties, and which may hopefully allow some forms of prediction of future events.

Model is reality scaled down and converted to a form we can comprehend.

A mathematical model is a model whose parts are mathematical concepts, such as constants, variables, functions, equations, inequalities etc.

Why we use modelling in agroforestry

In agro forestry tree component is retained for a long period of time (say 8 to 35 years or more). Now to develop growth and yield models, one has to harvest trees at different intervals to get a complete range of data. It

amounts to say that we will have to wait for years together to get the growth / yield information of the tree component. However, using the modelling tools we can predict the growth / yield of the tree component at a very early stage.

Materials and Methods

Description of study area

The experimental site is situated in Farm forestry unit, University of Agricultural Sciences, Dharwad. Dharwad is situated in Northern Transition Zone (zone 8) of Karnataka, with a latitude of 15⁰ 26¹ north, a longitude of 75⁰ 07¹ east and at an altitude of 678m above mean sea level. Total geographic area of Dharwad is 427329 ha. The annual rain fall in this zone is 749.48mm, major proportion of rainfall was received during June, July, August, September and October with heavy rains in July and September. The highest mean monthly maximum temperature recorded during May was 37.0⁰C which was 0.05⁰C higher as compared to the average maximum temperature of 56 years of same month. The lowest mean monthly temperature was recorded in December 13.1⁰C. Relative humidity was higher during June to September. The total number of rainy days (77days) was more than the average of previous 34 years (55.31 days).

Materials

The data for this study was procured from AICRP (All India Co-Ordinated Research Project on Agroforestry) UAS Dharwad. Experimental data contains information on parameters like grain yield of soybean at different distances from trees, height, dbh, crown area of various tree species of 14 years from 1993-2007 was used for the study. We tried different models such as linear regression model, quadratic model, cubic model,

logarithmic model, logistic model and exponential models and found that simple linear regression model, quadratic model were best fit based on R^2 values and standard error values

MMF model

The Morgan-Mercer-Flodin family function is nonlinear sigmoid growth functions. The general form of the Morgan-Mercer-Flodin family function (MMF) is

$$\text{MMF Model: } y = (a \cdot b + c \cdot x^d) / (b + x^d)$$

Where,

x is a independent variable.(dbh, height).
y is a dependent variable (age in years)

Where a, b, c, & d are the parameters in the model

Gompertz model

$$\text{Gompertz Relation: } y = a \cdot \exp(-\exp(b - cx))$$

Where

x is a independent variable.(dbh, height),
y is a dependent variable (age in years),
a, b, c re the parameters in the model

The Gompertz reliability growth model is often used when analyzing reliability data. It is most applicable when the data set follows a smooth curve. The Parameter Estimation for the Gompertz Models can be carried out by Using Least Squares in Nonlinear Regression as well as using linear regression methods.

Weibull model

$$\text{Weibull Model: } y = a - b \cdot \exp(-c \cdot x^d)$$

Where

x is a independent variable (dbh, height),
y is a dependent variable (age in years),
a, b c and d are the parameters in the model

Polynomial model

Polynomial regression is a form of linear regression in which the relationship between the independent variable x and the dependent variable y is modeled as an n th order polynomial. Polynomial regression fits a nonlinear relationship between the value of x and the corresponding conditional mean of y , denoted $E(y|x)$. In general, we can model the expected value of y as an n th order polynomial, yielding the general polynomial regression model,

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n + \varepsilon.$$

Where,

y is dependent variable
 a_i are Unknown parameters, $i = 0, 1, 2, \dots$,
x is independent variables.

Rational function

A rational function model is a generalization of the polynomial model. A rational function is basically a division of two polynomial functions. That is, it is a polynomial divided by another polynomial. In formal notation, a rational function would be symbolized like this:

$$f(x) = \frac{s(x)}{t(x)}$$

Where $s(x)$ and $t(x)$ are polynomial functions, and $t(x)$ cannot equal zero. A rational function is simply the ratio of two polynomial functions.

$$y = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_2 x^2 + b_1 x + b_0}$$

With n denoting a non-negative integer that defines the degree of the numerator and m is a non-negative integer that defines the degree of the denominator. For fitting rational function models, the constant term in the denominator is usually set to 1. Rational functions are typically identified by the degrees of the numerator and denominator.

Sinusoidal model

The sinusoidal speech model represents a speech signal as a linear combination of Sinusoids with time-varying parameters {amplitudes, frequencies, and phases}. A sinusoidal model to approximate a sequence Y_i is:

$$Y_i = C + \alpha \sin(\omega T_i + \phi) + E_i$$

Where, C is constant defining a mean level, α is an amplitude for the sine wave,

ω is the frequency, T_i is a time variable, ϕ is the phase, and E_i is the error sequence in approximating the sequence Y_i by the model.

Results and Discussion

The best fitted model for the height of the tree species *Ceiba pentandra*+ soybean, out of different models tried MMF model was proved better with ($R^2=0.9731$, $SE=35.82$),

followed by Gompertz relation with ($R^2=0.9721$, $SE=37.00$). For the fifth treatment combination *Terminalia bellerica*+ soybean, out of different models tried MMF model was proved better with ($R^2=0.9900$, $SE=32.85$), followed by Weibull model ($R^2=0.9900$, $SE=34.04$). For the sixth treatment combination *Mangifera indica*+ soybean, out of different models tried MMF model was proved better with ($R^2=0.9816$, $SE=35.37$), followed by polynomial ($R^2=0.9806$, $SE=35.85$) (Table 1).

The different models were tried to predicting yield of soybean with treatment combination *Ceiba pentandra*+ soybean, out of different models tried model Rational function was proved better with ($R^2=0.900$, $SE=1.965$), followed by Sinusoidal model with ($R^2=8.996$, $SE=1.975$). For the fifth treatment combination *Terminalia bellerica*+ soybean, Rational function was proved better with ($R^2=0.923$, $SE=1.529$), followed by Hoerl model ($R^2=0.884$, $SE=1.796$). For the sixth treatment combination *Mangifera indica*+ soybean, out of different models tried Hoerl model was proved better with ($R^2=0.937$, $SE=1.933$), followed by Rational function ($R^2=0.932$, $SE=2.66$) (Table 2).

Best suited models with their predicted height values of different tree species for next 6 years and the corresponding increment values.

Table.1 Prediction models for height growth of trees. (10 m x10m spacing, 10m between rows and 10m between trees)

Tree combination	Name of the model	MODEL	R ²	Standard error
<i>Ceiba pentandra</i> + soybean	Gompertz Model	$y=843.71 * \exp(-\exp(0.6833-0.4315x))$	0.9731**	35.82
	MMF	$y=(200.98*16.205+871.60*x^2.277)/(16.205+x^2.277)$	0.9721**	37.00
<i>Terminalia bellerica</i> + soybean	MMF	$y=(49.749*34.045+1052.29*x^2.169)/(34.045+x^2.169)$	0.9900**	32.85
	Weibull Model	$y=962.32-953.75 * \exp(-0.0663*x^1.5483)$	0.9900**	34.04
<i>Mangifera indica</i> + soybean	MMF	$y=(37.075*252.09+784.84*x^2.735)/(252.094+x^2.735)$	0.9816**	35.37
	Polynomial Model	$y=31.809+5.310x+11.294x^2+0.554x^3$	0.9806**	35.85

Table.2 Prediction models for yield of soybean crop at one meter distance from trees

Treatment	Name of the model	MODEL	R ²	Treatment
<i>Ceiba pentandra</i> + soybean	Rational function	$y=(2.914-0.1572x)/(1-0.2472x+0.0172x^2)$	0.900**	1.965
	Sinusoidal model	$y=7.596+6.951*\cos(0.508x-3.3055)$	8.996**	1.975
<i>Terminalia bellerica</i> + soybean	Rational function	$y=(-0.9097+1.68x)/(1-0.4114x+0.063x^2)$	0.923**	1.529
	Hoerl model	$y=3.612*(0.558^x)*(x^2.646)$	0.884**	1.796
<i>Mangifera indica</i> + soybean	Hoerl model	$y=2.13*(0.527^x)*(x^3.383)$	0.937**	1.933
	Rational function	$y=(2.030+0.684x)/(1-0.308x+0.031x^2)$	0.932**	2.66

Table.3 Table shows the best suited models with their predicted height values of different tree species for next 6 years and the corresponding increment values

Tree combination	Model name	model	X(yrs)	Y(cm)
<i>Ceiba pentandra</i> + soybean	Gompertz Model	$y=843.71 * \exp(-\exp(0.6833-0.4315x))$	15	823.74
			16	846.69
			17	865.54
			18	880.93
			19	893.45
	MMF	$y=(200.98*16.205+871.60*x^2.277)/(16.205+x^2.277)$	20	903.61
			15	835.43
			16	865.25
			17	891.70
			18	915.21
<i>Terminalia bellerica</i> + soybean	MMF	$y=(49.749*34.045+1052.29*x^2.169)/(34.045+x^2.169)$	19	936.14
			20	954.82
			15	964.68
			16	975.25
			17	984.10
	Weibull Model	$y=962.32-953.75*\exp(-0.0663*x^1.5483)$	18	991.57
			19	997.57
			20	1003.3
			15	950.49
			16	954.86
<i>Mangifera indica</i> + soybean	MMF	$y=(37.075*252.09+784.84*x^2.735)/(252.094+x^2.735)$	17	957.69
			18	959.49
			19	960.62
			20	961.31
			15	685.56
	Polynomial Model	$y=31.809-5.310x+11.294x^2-0.554x^3$	16	699.80
			17	711.52
			18	721.23
			19	729.32
			20	736.12
			15	602.60
			16	637.11
			17	654.33
			18	650.92
			19	623.56
			20	630.05

Table.4 Table shows the best suited models with their predicted yield values of soybean at one meter distance from the tree for next 6 years and the corresponding increment values

<i>Ceiba pentandra</i> + <i>soybean</i>	Rational function	$y=(2.914-0.1572x)/(1-0.2472x+0.0172x^2)$	15	0.479
			16	0.275
			17	0.136
			18	0.039
			19	0.029
			20	0.078
	Sinusoidal model	$y=7.596+6.951*\cos (0.508x-3.3055)$	15	4.993
			16	8.463
			17	11.71
			18	13.92
			19	14.52
			20	13.37
<i>Terminalia bellerica</i> + <i>soybean</i>	Rational function	$y=(-0.9097+1.68x)/(1-0.4114x+0.063x^2)$	15	2.697
			16	2.446
			17	2.249
			18	2.080
			19	1.935
			20	1.808
	Hoerl model	$y=3.612*(0.558^x)*(x^2.646)$	15	0.728
			16	0.481
			17	1.315
			18	0.204
			19	0.131
			20	0.084
<i>Mangifera indica</i> + <i>soybean</i>	Hoerl model	$y=2.13*(0.527^x)*(x^3.383)$	15	1.366
			16	0.895
			17	0.579
			18	0.370
			19	0.234
			20	0.146
	Rational function	$y=(2.030+0.684x)/(1-0.308x+0.031x^2)$	15	3.462
			16	3.066
			17	2.745
			18	2.480
			19	2.258
			20	2.069

In case on *Ceiba pentandra* + soybean, the best selected models are Gompertz and MMF model. In case of Gompertz model, the increment seen

in height between 15th to 20th year ranges between 823.74cm to 903.61cm, whereas in case of MMF model it ranges between

835.43cm to 954.82cm. In case on *Terminalia bellerica*+ soybean, the best selected models are MMF and Weibull model. In case of MMF model, the increment seen in height between 15th to 20th year ranges between 964.68cm to 1003.3cm, whereas in case of Weibull model it ranges between 950.49cm to 961.31cm.

In case on *Mangifera indica* + soybean, the best selected models are MMF and Polynomial model. In case of MMF model, the increment seen in height between 15th and 20th year ranges between 685.56cm and 736.12cm, whereas in case of Polynomial model it ranges between and 602.60cm and 623.56cm (Table 3).

Best suited models with their predicted yield values of soybean for next 6 years and the corresponding decreased values at one meter distance from the trees.

In case on *Ceiba pentandra* + soybean, the best selected models are Rational function and Sinusoidal model. In case of Rational function the decline as been seen in yield between 15th to 20th year ranges between 0.479(kg/ha) to -0.0783(kg/ha). Whereas in case of Sinusoidal model 4.993(kg/ha) to 13.37(kg/ha).

In case on *Terminalia bellerica*+ soybean, the best selected models are Rational function and Hoerl model. In case of Rational function, the decline as been seen in yield between 15th to 20th year ranges between 2.679(kg/ha) to 1.808(kg/ha). Whereas in case of Hoerl model 0.728(kg/ha) to 0.084 (kg/ha). In case on *Mangifera indica*+ soybean, the best selected models are Hoerl model and Rational function. In case of Hoerl model the decline as been seen in yield between 15th and 20th year ranges between 1.366 (kg/ha) and 0.146 (kg/ha).

Whereas in case of Rational function 3.462(kg/ha) to 2.069 (kg/ha) (Table 4).

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