Int.J.Curr.Microbiol.App.Sci (2018) *7*(10): 3358-3362

International Journal of Current Microbiology and Applied Sciences
ISSN: 2319-7706 **Volume 7 Number 10 (2018)**
Journal homepage: http://www.ijcmas.com

Original Research Article https://doi.org/10.20546/ijcmas.2018.710.389

Impact of Robust Estimators on Variance Estimation in Survey Sampling, Using Conventional and Non-Conventional Parameters as Auxiliary Information

M.A. Bhat*, T.A. Raja and S. Maqbool

Division of Agricultural Economics and Statist SKUAST-Kashmir (190025), India

**Corresponding author*

A B S T R A C T

Keywords

Sample Random Sampling, Bias, MSE, Quartiles and efficiency

Article Info

Accepted:
24 September 2018
Available Online:
10 October 2018

In the present study, we have developed new estimators for the estimation of finite population variance by using auxiliary information as combination of conventional and non-conventional measures. Bias and mean square error has been worked out up to the first order of approximation. The empirical study has been carried out through numerical demonstration, under which improved estimators have performed better than the other existing estimators.

Introduction

Variance estimation has great scope in the fields like medical sciences, Agricultural and Horticultural sciences, Biological sciences and Industries etc. In the field of medical sciences, a physician needs to have the knowledge about the levels of variations like pulse rate, body temperature and blood pressure to prescribe the suitable treatment. Similarly an agriculturalist needs the knowledge about the levels of variations like soil fertility variation, genetic variations and climatic variations to plant the crop. Thus variations are present everywhere in our day to day life. In order to obtain the precise and valid estimates, we have proposed new estimators for the estimation of finite population variance. Various authors have generalized many estimators such as Isaki (1983), who introduced the ratio and regression estimator, Kadilar and Cingi (2006a), who searched the estimators by utilizing the coefficient of skewness as supplementary information to enhance the efficiency of estimator. Upadhya and Singh (1999) incorporated the coefficient of kurtosis as auxiliary information to improve the efficiency of estimators over existing estimators. Similarly Bhat *et al.,* (2018) have used linear combination of skewness and quartiles as auxiliary information to obtain the precision of estimators. Sarginder Singh (2003), Advance Sampling theory with Applications (2003), page (1135-1136).

In our present study, we have incorporated the combination of conventional and non-conventional parameters to accelerate the efficiency of proposed estimators.

Let the finite population under survey be $U = \{U_1, U_2, ..., U_N\}$, consists of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i, i = 1,2,3........N$, giving a vector $Y = \{y_1, y_2, ..., y_N\}$. The goal is to estimate the populations mean $\bar{Y} = \frac{1}{N}\sum_{I=1}^{N} y_i$ or its variance $S_Y^2 = \frac{1}{N-1}\sum_{I=1}^{N}(y_i - \bar{y})^2$ on the basis of random sample selected from a population U. In this paper, our aim is to estimate the precise and reliable estimates for finite population variance when the population under survey is non-normal or skewed.

Materials and Methods

Notations

$N = population\ size$, $n = sample\ size$.

$\gamma = \frac{N-n}{Nn}$

$Y = Study$ Variable, X = Auxiliary Variable,

$\bar{X}, \bar{Y} = population\ means$,

$\bar{x}\ , \bar{y} = Sample\ means$,

$S_Y^2\ , S_x^2 = population\ Variances$,

$s_y^2\ , s_x^2 = Sample\ variances$,

$C_x\ , C_y = Cofficient\ of\ variations$,

$\rho = Cofficient\ of\ Correlation$,

$\beta_{1(x)} = Cofficient\ of\ Skewness$,

$\beta_{2(x)} = Cofficient\ of\ kurtosis$,

$\beta_{2(y)} = Cofficient\ of\ kurtosis$,

$Q_1 = first\ quartile$, $Q_2 = second\ quartile$, $Q_a = quartile\ average$, $Q_d = quartile\ deviation$ and $TM = Tri-mean$, $\lambda_{rs} = \frac{\mu_{rs}}{\mu_{rs}^{\frac{r}{2}}\mu_{rs}^{\frac{s}{2}}}$, $\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(Y-\bar{Y})^r (X-\bar{X})^s$

Existing Estimators from the literature

Ratio type Variance estimator proposed by Isaki (1983)

$$\hat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2}$$

Bias ($(\hat{S}_R^2) = \gamma S_y^2\ [(\beta_{2(x)} - 1) - (\lambda_{22} - 1)]$

MSE

$(\hat{S}_R^2) =$

$\gamma S_y^4\ [(\beta_{2(y)} - 1) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)]$

Ratio type Variance estimator proposed by Kadilar and Cingi (2006a)

$$\hat{S}_{kc1}^2 = s_y^2 \left[\frac{S_x^2 + C_x}{s_x^2 + C_x}\right]$$

Bias

$(\hat{S}_{kc1}^2) = \gamma S_y^2 A_1\ [A_1(\beta_{2(x)} - 1) - (\lambda_{22} - 1)]$

MSE

$(\hat{S}_{kc1}^2) =$

$\gamma S_y^4\ [(\beta_{2(y)} - 1) + A_1^2\ (\beta_{2(x)} - 1) - 2A_1(\lambda_{22} - 1)]$

Ratio type variance estimator proposed by Upadhya and Singh (1999)

$$\hat{S}_{US}^2 = s_y^2 \left[\frac{S_x^2 + \beta_{2x}}{s_x^2 + \beta_{2x}}\right]$$

Bias

$(\hat{S}_{US}^2) = \gamma S_y^2 A_{US}\ [A_{US}(\beta_{2(x)} - 1) - (\lambda_{22} - 1)]$

MSE $(\hat{S}^2_{US}) = \gamma S_y^4 \left[(\beta_{2(y)}-1) + A^2_{US}(\beta_{2(x)}-1) - 2A_{US}(\lambda_{22}-1)\right]$

New improved estimators

$$\hat{S}^2_{MS_1} = s_y^2\left[\frac{S_x^2 + (\bar{X}\times TM)}{s_x^2 + (\bar{X}\times TM)}\right],$$

$$\hat{S}^2_{MS_2} = s_y^2\left[\frac{S_x^2 + (\bar{X}\times Q_d)}{s_x^2 + (\bar{X}\times Q_d)}\right]$$

We have derived here the bias and mean square error of the proposed estimator $\hat{S}^2_{MSi};\ i=1,2$ to first order of approximation as given below

Let $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$. Further we can write $s_y^2 = S_y^2(1+e_0)$ and $s_x^2 = S_x^2(1+e_1)$ and from the definition of e_0 and e_1 we obtain:

$E[e_0] = E[e_1] = 0$, $E[e_0^2] = \frac{1-f}{n}(\beta_{2(y)}-1)$,

$E[e_1^2] = \frac{1-f}{n}(\beta_{2(x)}-1)$,

$E[e_0e_1] = \frac{1-f}{n}(\lambda_{21}-1)$

The proposed estimator $\hat{S}^2_{MSi};\ i=1,2,3,r,d,a$ is given below:

$$\hat{S}^2_{MSi} = s_y^2\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + \alpha a_i}\right] \quad (5.1)$$

$$\Rightarrow \quad \hat{S}^2_{MSi} = s_y^2(1+e_0)\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + e_1 S_x^2 + \alpha a_i}\right] \quad \Rightarrow$$

$$\hat{S}^2_{MSi} = \frac{S_y^2(1+e_0)}{(1+A_{MSi}e_1)}$$

Where,

$A_{MSi} = \frac{S_x^2}{S_x^2 + \alpha a_i}$,

$a_i = (\bar{X}\times TM), (\bar{X}\times Q_d);\ i = 1,2$

$$\Rightarrow \hat{S}^2_{MSi} = S_y^2(1+e_0)(1+A_{MSi}e_1)^{-1} \quad (5.2)$$

$$\Rightarrow \hat{S}^2_{MSi} = S_y^2(1+e_0)(1 - A_{MSi}e_1 + A^2_{MSi}e_1^2 - A^3_{MSi}e_1^3 + ...) \quad (5.3)$$

Expanding and neglecting the terms more than 3rd order, we get

$$\hat{S}^2_{MSi} = S_y^2 + S_y^2e_0 - S_y^2A_{MSi}e_1 - S_y^2A_{MSi}e_0e_1 + S_y^2A^2_{MSi}e_1^2 \quad (5.4)$$

$$\hat{S}^2_{MSi} - S_y^2 = S_y^2e_0 - S_y^2A_{MSi}e_1 - S_y^2A_{MSi}e_0e_1 + S_y^2A^2_{MSi}e_1^2 \quad (5.5)$$

By taking expectation on both sides of (5.5), we get

$$E(\hat{S}^2_{MSi} - S_y^2) = S_y^2E(e_0) - S_y^2A_{MSi}E(e_1) - S_y^2A_{MSi}E(e_0e_1) + S_y^2A^2_{MSi}E(e_1^2) \quad (5.6)$$

$$Bias(\hat{S}^2_{MSi}) = S_y^2A^2_{MSi}E(e_1^2) - S_y^2A_{MSi}E(e_0e_1) \quad (5.7)$$

$$Bias(\hat{S}^2_{MSi}) = \gamma S_y^2A_{MSi}[A_{MSi}(\beta_{2(x)}-1) - (\lambda_{21}-1)] \quad (5.8)$$

Squaring both sides of (5.5) and neglecting the terms more than 2nd order and taking expectation, we get

$$E(\hat{S}^2_{MSi} - S_y^2)^2 = S_y^4E(e_0^2) + S_y^4A^2_{MSi}E(e_1^2) - 2S_y^4A_{MSi}E(e_0e_1) \quad (5.9)$$

$$MSE(\hat{S}^2_{MSi}) = \gamma S_y^4[(\beta_{2(y)} - 1) + A^2_{MSi}(\beta_{2(x)} - 1) - 2A_{MSi}(\lambda_{21} - 1)]$$
(5.10)

Efficiency conditions

Here, we have derived the efficiency conditions of proposed estimators with other existing estimators under which proposed estimators have performed better than the existing estimators

The bias and Mean square error of existing ratio type estimators up to the first order of approximation is given by

$$Bias(\hat{S}^2_K) = \gamma S_y^2 R_K [R_K(\beta_{2x} - 1) - (\lambda_{21} - 1)]$$
(6.1)

$$MSE(\hat{S}^2_K) = \gamma S_y^4 [(\beta_{2y} - 1) + R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{21} - 1)]$$
(6.2)

$R_K = Existing.constant$
$, K = 1,2,3,4........$

Bias, MSE and constant of proposed estimators is given by

$$Bias(\hat{S}^2_P) = \gamma S_y^2 R_P [R_P(\beta_{2x} - 1) - (\lambda_{21} - 1)] \quad (6.3)$$

$$MSE(\hat{S}^2_P) = \gamma S_y^4 [(\beta_{2y} - 1) + R_P^2(\beta_{2x} - 1) - 2R_P(\lambda_{21} - 1)]$$
(6.4)

$R_P = proposed.constant$
$P = 1,2,3.............$

From Equation (6.2) and (6.3), we have

$$MSE(\hat{S}^2_P) \leq MSE(\hat{S}^2_K) if \lambda_{21} \geq 1 + \frac{(R_P + R_K)(\beta_{2x} - 1)}{2}$$

$$MSE(\hat{S}^2_P) \leq MSE(\hat{S}^2_K)$$

$$\gamma S_y^4 [(\beta_{2y} - 1) + R_P^2(\beta_{2x} - 1) - 2R_P(\lambda_{21} - 1)] \leq$$
$$\gamma S_y^4 [(\beta_{2y} - 1) + R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{21} - 1)]$$
(6.5)

$$\Rightarrow [(\beta_{2y} - 1) + R_P^2(\beta_{2x} - 1) - 2R_P(\lambda_{21} - 1)] \leq [(\beta_{2y} - 1) + R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{21} - 1)] \quad (6.6)$$

$$\Rightarrow [R_P^2(\beta_{2x} - 1) - 2R_P(\lambda_{21} - 1)] \leq [R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{21} - 1)] \quad (6.7)$$

$$\Rightarrow (\beta_{2x} - 1)(R_P^2 - R_K^2)[-2R_P(\lambda_{21} - 1)] \leq [-2R_K(\lambda_{21} - 1)] \quad (6.8)$$

$$\Rightarrow (\beta_{2x} - 1)(R_P^2 - R_K^2)[-2(\lambda_{21} - 1)(R_P - R_K)] \leq 0 \quad (6.9)$$

$$\Rightarrow (\beta_{2x} - 1)(R_P^2 - R_K^2) \leq [2(\lambda_{21} - 1)(R_P - R_K)]$$
(6.10)

$$\Rightarrow (\beta_{2x} - 1) \leq \frac{2(\lambda_{21} - 1)(R_P - R_K)}{(R_P^2 - R_K^2)} \quad (6.11)$$

$$\Rightarrow (\beta_{2x} - 1) \leq \frac{2(\lambda_{21} - 1)(R_P - R_K)}{(R_P - R_K)(R_P + R_K)} \quad (6.12)$$

$$\Rightarrow (\beta_{2x} - 1)(R_P + R_K) \leq 2(\lambda_{21} - 1) \quad (6.13)$$

By solving equation (6.13), we get

$$MSE(\hat{S}^2_P) \leq MSE(\hat{S}^2_K) if \lambda_{21} \geq 1 + \frac{(R_P + R_K)(\beta_{2x} - 1)}{2}$$

Numerical Illustration

We use the data set of Sarginder Singh; Advance Sampling theory with Applications, page (1135-1136) (source: Agricultural Statistics, 1999 Washington, US) in which Amount (in $000) is real estate farm loans in different states during 1997 denoted by X (auxiliary variable) and Amount (in $000) is non-real estate farm loans in different states during 1997 denoted by Y (study variable).

Table.1 Bias and mean square error of the existing and the proposed estimators

Estimators	Bias	MSE
Isaki (1983)	10162.12	2154636192.33
Kadilar and Cingi (2006a)	10142.43	2147538179.27
Upadhya and Singh (1999)	10142.43	2147538179.27
Proposed (MS1)	5539.36	515446446.37
Proposed (MS2)	5083.60	352496052.61

Table.2 Percent relative efficiency of proposed estimators with existing estimators

Existing Estimators	Proposed estimator P_1	Proposed Estimator P_2
Isaki (1983)	418.01	611.25
Kadilar & Cingi (2006a)	416.63	609.23
Upadhya and Singh (1999)	416.63	609.23

We apply the proposed and existing estimators to this data set and the data statistics is given below:

N=50, S_x=1084.48, n=20, Cx=1.2357, $\bar{X}$ = 877.558, $\beta_{2(x)}$ = 1.9291, $\bar{Y}$ = 555.434, $\beta_{2(y)}$ =0.856227 ρ =0.804 , $\beta_{1(x)}$ =1.6624 λ_{21} =0.9387 S_y=584.825, Q_2=452.51, Q1=57.37, TM = 493.7, Q3= 1215.67, Q_d= 579.15, Q_a= 636.52, Q_r= 1158.30.

Results and Discussion

The modified new improved estimators by utilizing combination of conventional and non-conventional parameters as auxiliary information have performed better than other existing estimators. The improvement can be easily judged from the table 1 and 2 by comparing the bias, MSE and percent relative efficiency of existing and new modified estimators. Hence new modified estimators may be preferred over existing estimators for use in practical applications.

References

Isaki, C.T. (1983). Variance estimation using auxiliary information. *Journal of the American Statistical Association, 78, 117-123.*

Kadilar, C. and Cingi, H. (2006a). Improvement in Variance estimation using auxiliary information Hacettepe *Journal of mathematics and Statistics, 35(1). 117-115.*

Upadhya, L. N. and Singh, H. P, (1999): Use of auxiliary variable in the estimation of population variance, *Mathematical forum, 4, 33-36 (1936).*

Bhat M.A., S. Maqbool, S.A. Saraf, Ab. Rouf and S.H. Malik, NLMID: 101666096. *Journal of Advances in Research 13(2): 1-6, 2018 Article no. 37321.*

Sarginder Singh; *Advance Sampling theory with Applications (2003), page (1135-1136). Publisher; Springer-Science, business.*

How to cite this article:

Bhat, M.A., T.A. Raja and Maqbool, S. 2018. Impact of Robust Estimators on Variance Estimation in Survey Sampling, Using Conventional and Non-Conventional Parameters as Auxiliary Information. *Int.J.Curr.Microbiol.App.Sci.* 7(10): 3358-3362.
doi: https://doi.org/10.20546/ijcmas.2018.710.389